MathExcel Monday Worksheet K:
Exam III Review

1. Find the average value of the following functions:

   (a) $f(x) = \dfrac{3}{x^2+1}$ on [0,1].

   (b) $h(x) = \dfrac{\sin(\pi/x)}{x^2}$ on [1,2].

2. (a) State the Mean Value Theorem for Integrals.

   (b) Find $f_{avg}[0,7]$ when $f(x) = 2 + 6x - 3x^2$.

   (c) Must there be a value of c such that $f_{ave}[0,7] = f(c)$? Justify your answer. If you said yes, find such a c.

3. (a) Suppose that $f(x) > g(x)$ for all x on the interval $[a, b]$. Write a formula for the volume of the solid obtained by rotating the region bounded by $f(x), g(x), x = a$, and $x = b$ about the x-axis using the disc/washer method.

   (b) Suppose that $f(x)$ is a function that increases on the interval $[0, b]$. Use the method of cylindrical shells to write a formula for the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = f(x)$ and $y = f(b)$ about the line $y = -1$.

4. (a) Find the volume of the solid whose base is the circle $x^2 + y^2 = 2^2$ and the cross sections perpendicular to the x-axis are squares.

   (b) Find the volume of the solid whose base is the area bounded by the inequalities $0 \leq x \leq 2$ and $0 \leq y \leq 3$, where cross sections perpendicular to the y-axis are semicircles.

   (c) Consider the region inside the circle $x^2 + y^2 = 9$ and above the line $y = 1$. Find the volume of the solid given by revolving this region around the x-axis.

   (d) Use the shell method to find the volume of the solid given by revolving the region between the graph of $y = -x^2 + 6x - 8$ and $y = 0$ around the y-axis.

   (e) The region bounded by $y = \sqrt{x}$, $y = 1$, and $x = 0$ is rotated about the y-axis to form a solid.

      i. Draw a clear picture of the region described by the three equations.

      ii. Write an integral that uses the disk method to compute the volume.

      iii. Write an integral that uses the shell method to compute the volume.

      iv. Select one of the above integrals and evaluate it to compute the volume.

5. Arc length and surface area:

   (a) Find the length of the arc $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ over the interval $[1, 2e]$.

   (b) Find the area of the surface of revolution when the arc $y = \sin x$ over the interval $[0, \pi]$ is rotated about the x-axis.

6. Center of mass:

   (a) Find the center of mass for the system of particles of masses $4, 2, 5, 1$ located at $(1, 2)$, $(-3, 2)$, $(2, -1)$, $(4, 0)$.

   (b) Find the centroid of the region under the graph $y = 9 - x^2$ over the interval $[0, 3]$.

7. Parametric equations:

   (a) The motion of a particle in the plane obeys the parametric equations $x(t) = 2\cos t + 5$ and $y(t) = 3\sin t + 1$ for $0 \leq t \leq \pi$. Sketch the path of the particle and indicate the direction of travel.

   (b) Compute $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for the parametrically defined curve $x(t) = t + e^t$ and $y(t) = e^{-t}$.

   (c) Consider the parametrized curve $x(t) = t^3 - 6t^2 + 6$, $y(t) = 2t^2 + 8t + 10$, where $-\infty < t < \infty$.

      i. Find the value(s) of t that corresponds to the point $(-1, 4)$.

      ii. Find all points on the curve where the curve has a horizontal tangent line.

      iii. Find all points on the curve where the curve has a vertical tangent line.

   (d) For the following parametric curve, find an equation for the tangent line to the curve $c(t) = (e^{\sqrt{t}}, t - \ln(t^2))$ at $t = 1$.